

Critical phenomenon of the near room temperature skyrmion material FeGe

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Abstract

The cubic B20 compound FeGe, which exhibits a near room temperature skyrmion phase, is one of the most promising candidate of the next generation spintronic devices. In this work, the critical behavior of the cubic FeGe is investigated by means of bulk dc-magnetization. We obtain the critical exponents ($\beta = 0.336 \pm 0.004$, $\gamma = 1.352 \pm 0.003$, and $\delta = 5.267 \pm 0.001$), where the self-consistency and reliability are verified by the Widom scaling law and scaling equations. The magnetic exchange distance is found to decay as $J(r) \approx r^{-4.9}$, which is close to the theoretical prediction of 3D-Heisenberg model (r^{-5}). The critical behavior of FeGe indicates a short-range magnetic interaction. Meanwhile, the critical exponents also imply an anisotropic magnetic coupling in this system.

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In recently years, skyrmion state, which is a topologically protected nanoscale vortex-like spin structure, has attracted great interest due to its potential application in spintronic storage function [1–12]. However, the skyrmion phase usually exists in narrow magnetic field (H) and low temperature (T) regimes, which fundamentally limits its application [13]. Therefore, room-temperature skyrmion materials hosting stable skrymion phase are paid considerable attention [14]. The cubic FeGe belongs to the space group $P2_13$, in which the non-centrosymmetric cell results in a weak Dzyaloshinskii-Moriya (DM) interaction. The competition of DM interaction between the much stronger ferromagnetic exchange finally causes a long modulation period of a helimagnetic ground state [1, 2, 15]. A bulk FeGe sample exhibits a long-range magnetic order at Curie temperature $T_C = 278.2$ K, and displays a complex succession of temperature-driven crossovers in the vicinity of T_C [16, 17]. The skyrmion phase emerges in a narrow temperature range just below T_C in the filed range from 0.15 to 0.4 kOe. The existence of the near room temperature skyrmion phase in FeGe, to our knowledge the highest T_C in B20 skyrmion compounds, makes it one of the most promising candidate of the next generation spintronic devices. Recently, more stable skyrmion phase has been realized in FeGe thin film, and it has been claimed that the skyrmions can be tuned by the crystal lattice [18–20]. These findings pave a new path to design quantum-effect devices based on the tunable skyrmion dynamics. On the other hand, multiple and complex magnetic interactions have also been found in FeGe. An inhomogeneous helimagnetic state has been discovered above T_C due to the strong precursor phenomena [16, 21]. More interestingly, it has been revealed that the helical axis (q -vector direction) orientates depending on temperature. At zero magnetic field, the helical axis is along the $<100>$ direction below 280 K. With decreasing temperature, it changes to the $<111>$ direction at 211 K [17].

In view of the potential application and abundant physics in FeGe, a deep investigation of its magnetic exchange is necessary. In this work, the critical behavior of FeGe has been investigated by means of bulk dc-magnetization. The critical exponents ($\beta = 0.336 \pm 0.004$, $\gamma = 1.352 \pm 0.003$, and $\delta = 5.267 \pm 0.001$) are obtained, where the self-consistency and reliability are verified by the Widom scaling law and the scaling equations. These critical behavior of FeGe indicates a short-range magnetic interaction with a magnetic exchange distance decaying as $J(r) \approx r^{-4.9}$. The obtained critical exponents also imply an anisotropic magnetic coupling in FeGe system.

I. RESULTS AND DISCUSSION

It is well known that the critical behavior for a second-order phase transition can be investigated through a series of critical exponents. In the vicinity of the critical point, the divergence of correlation length ξ leads to universal scaling laws for the spontaneous magnetization M_S and initial susceptibility χ_0 . Subsequently, the mathematical definitions of the exponents from magnetization are described as [22, 23]:

$$M_S(T) = M_0(-\varepsilon)^\beta, \varepsilon < 0, T < T_C \quad (1)$$

$$\chi_0^{-1}(T) = (h_0/M_0)\varepsilon^\gamma, \varepsilon > 0, T > T_C \quad (2)$$

$$M = DH^{1/\delta}, \varepsilon = 0, T = T_C \quad (3)$$

where $\varepsilon = (T - T_C)/T_C$ is the reduced temperature; M_0/h_0 and D are the critical amplitudes. The parameters β (associated with M_S), γ (associated with χ_0), and δ (associated with T_C) are the critical exponents. Universally, in the asymptotic critical region ($|\varepsilon| < 0.1$), these critical exponents should follow the Arrott-Noakes equation of state [24]:

$$(H/M)^{1/\gamma} = (T - T_C)/T_C + (M/M_1)^{1/\beta} \quad (4)$$

Therefore, the critical exponents β and γ can be obtained by fitting the $M_S(T)$ and $\chi_0^{-1}(T)$ curves using the modified Arrott plot of $M^{1/\beta}$ vs $(H/M)^{1/\gamma}$. Meanwhile, δ can be generated directly by the $M(H)$ at the critical temperature T_C according to Eq. (3).

Generally, the critical temperature T_C can be roughly determined by the temperature dependence of magnetization [$M(T)$]. Figure 1 (a) shows the $M(T)$ curves for FeGe under zero-field-cooling (ZFC) and field-cooling (FC) with an applied field $H = 100$ Oe. The $M(T)$ curves exhibit an abrupt decline with the increase of temperature, corresponding to the helimagnetic-paramagnetic (HM-PM) transition. A sharp peak is observed at $T = 278.5$ K. The inset of Fig. 1 (a) gives dM/dT vs T , where $T_C \approx 283$ K is determined from the minimum of the dM/dT curve. Wilhelm *et al.* has demonstrated that a long-rang magnetic order occurs below 278.2 K, however, an inhomogeneous helical state has existed above that temperature due to the strong precursor phenomena [16]. The higher T_C determined here indicates the appearance of precursor phenomena which may be caused by the strong spin fluctuation [21]. Figure 1 (b) shows the isothermal magnetization $M(H)$ at 4 K, which exhibits a typical magnetic ordering behavior. The inset of Fig. 1 (b) plot the magnified

$M(H)$ in lower field regime, which shows that the saturation field $H_S \approx 3000$ Oe. No magnetic hysteresis is found on the $M(H)$ curve, indicating no coercive force for FeGe.

Usually, the critical exponents can be determined by the Arrott plot. For the Landau mean-field model with $\beta = 0.5$ and $\gamma = 1.0$ [25], the Arrott-Noakes equation of state evolves into $H/M = A + BM^2$, the so called Arrott equation. In order to construct an Arrott plot, the isothermal magnetization curves $M(H)$ around T_C are measured as shown in Fig. 2 (a). The Arrott plot of M^2 vs H/M for FeGe is depicted in Fig. 2 (b). According to the Banerjee's criterion, the slope of line in the Arrott plot indicates the order of the phase transition: negative slope corresponds to first-order transition while positive to second-order one [26]. Therefore, the Arrott plot of FeGe implies a second-order phase transition. According to the Arrott plot, the M^2 vs H/M generally present a series of parallel straight lines around T_C , where H/M vs. M^2 at T_C just pass through the origin [27]. One can see that all M^2 vs H/M curves show quasi-straight lines with positive slopes in high field range. However, all lines show an upward curvature and are not parallel to each other, indicating that the $\beta = 0.5$ and $\gamma = 1.0$ within the framework of Landau mean-field model is unsatisfied. Therefore, a modified Arrott plot should be employed.

Four kinds of possible exponents belonging to the 3D-Heisenberg model ($\beta = 0.365$, $\gamma = 1.336$), 3D-Ising model ($\beta = 0.325$, $\gamma = 1.24$), 3D-XY model ($\beta = 0.345$, $\gamma = 1.316$), and tricritical mean-field model ($\beta = 0.25$, $\gamma = 1.0$) [25, 28] are used to construct the modified Arrott plots, as shown in Figs. 3 (a), (b), (c), and (d). All these four constructions exhibit quasi-straight lines in the high field region [29–31]. Apparently, the lines in Fig. 3 (d) are not parallel to each other, indicating that the tricritical mean-field model is not satisfied. However, all lines in Figs. 3 (a), (b) and (c) are almost parallel to each other. To determine an appropriate model, the modified Arrott plots should be a series of parallel lines in the high field region with the same slope, where the slope is defined as $S(T) = dM^{1/\beta}/d(H/M)^{1/\gamma}$. The normalized slope (NS) is defined as $NS = S(T)/S(T_C)$, which enables us to identify the most suitable model by comparing the NS with the ideal value of '1' [29]. Plots of NS vs T for the four different models are shown in Fig. 4. One can see that the NS of 3D-Heisenberg model is close to '1' mostly above T_C , while that of 3D-Ising model is the best below T_C . This result indicates that the critical behavior of FeGe may not belong to a single universality class.

The precise critical exponents β and γ should be achieved by the iteration method [32].

The linear extrapolation from the high field region to the intercepts with the axes $M^{1/\beta}$ and $(H/M)^{1/\gamma}$ yields reliable values of $M_S(T, 0)$ and $\chi_0^{-1}(T, 0)$, which are plotted as a function of temperature in Fig. 5 (a). By fitting to Eqs. (1) and (2), one obtains a set of β and γ . The obtained β and γ are used to reconstruct a new modified Arrott plot. Consequently, new $M_S(T, 0)$ and $\chi_0^{-1}(T, 0)$ are generated from the linear extrapolation from the high field region. Therefore, another set of β and γ can be yielded. This procedure has been repeated until β and γ did not change. As one can see, the obtained critical exponents by this method are independent on the initial parameters, which confirms these critical exponents are reliable and intrinsic. In this way, it is obtained that $\beta = 0.336 \pm 0.004$ with $T_C = 283.18 \pm 0.05$ and $\gamma = 1.352 \pm 0.003$ with $T_C = 282.87 \pm 0.08$ for FeGe. The critical temperature T_C from the modified Arrott plot is in agreement with that obtained from the derivative $M(T)$ curve, indicating strong critical fluctuation in FeGe [21].

Figure 5 (b) shows the isothermal magnetization $M(H)$ at the critical temperature $T_C = 283$ K, with the inset plotted on a $\lg - \lg$ scale. One can see that the $M(H)$ at T_C exhibits a straight line on a $\lg - \lg$ scale for $H > H_S$. We determine that $\delta = 5.297 \pm 0.001$ in the high field region ($H > H_S$). According to the statistical theory, these critical exponents should fulfill the Widom scaling law [33]:

$$\delta = 1 + \frac{\gamma}{\beta} \quad (5)$$

As a result, $\delta = 5.024 \pm 0.005$ is calculated according to the Widom scaling law, in agreement with the results from the experimental critical isothermal analysis. The self-consistency of the critical exponents demonstrates that they are reliable and unambiguous.

Finally, these critical exponents should obey the scaling equations. Two different constructions have been used in this work, both of which are based on the scaling equations of state. According to the scaling equations, in the asymptotic critical region, the magnetic equation is written as [22]:

$$M(H, \varepsilon) = \varepsilon^\beta f_\pm(H/\varepsilon^{\beta+\gamma}) \quad (6)$$

where f_\pm are regular functions denoted as f_+ for $T > T_C$ and f_- for $T < T_C$. Defining the renormalized magnetization as $m \equiv \varepsilon^{-\beta} M(H, \varepsilon)$, and the renormalized field as $h \equiv H \varepsilon^{-(\beta+\gamma)}$, the scaling equation indicates that m vs h forms two universal curves for $T > T_C$ and $T < T_C$ respectively [34, 35]. Based on the scaling equation [$m = f_\pm(h)$], the isothermal magnetization around T_C for FeGe is replotted in Fig. 6 (a), where all experimental data

collapse onto two universal branches. The inset of Fig. 6 (a) shows the m^2 vs h/m , where all $M - T - H$ curves should collapse onto two independent universal curves. In addition, the scaling equation of state takes another form [22, 34]:

$$\frac{H}{M^\delta} = k\left(\frac{\varepsilon}{H^{1/\beta}}\right) \quad (7)$$

where $k(x)$ is the scaling function. Based on Eq. (7), all experimental curves will collapse onto a single curve. Figure 6 (b) shows the $MH^{-1/\delta}$ vs $\varepsilon H^{-1/(\beta+\gamma)}$ for FeGe, where the experimental data collapse onto a single curve, and T_C locates at the zero point of the horizontal axis. The well-rescaled curves further confirm the reliability of the obtained critical exponents.

The obtained critical exponents of FeGe and other related materials, as well as those from different theoretical models are summarized in Table I for comparison. One can see that the critical exponent γ of FeGe is close to that of 3D-Heisenberg model, while β approaches to that of 3D-Ising or 3D-XY mode, indicating that the critical behavior of FeGe do not belong to a single universality class. Anyhow, all these three models indicate a short-range magnetic coupling, implying the existence of short-range magnetic interaction in FeGe. As we know, for a homogeneous magnet, the universality class of the magnetic phase transition depends on the exchange distance $J(r)$. M. E. Fisher *et al.* have treated this kind of magnetic ordering as an attractive interaction of spins, where a renormalization group theory analysis suggests $J(r)$ decays with distance r as [36, 37]:

$$J(r) \approx r^{-(d+\sigma)} \quad (8)$$

where d is the spatial dimensionality and σ is a positive constant. Moreover, there is [36, 38]:

$$\gamma \approx 1 + \frac{4}{d} \frac{n+2}{n+8} \Delta\sigma + \frac{8(n+2)(n-4)}{d^2(n+8)^2} \left[1 + \frac{2G(\frac{d}{2})(7n+20)}{(n-4)(n+8)} \right] \Delta\sigma^2 \quad (9)$$

where $\Delta\sigma = (\sigma - \frac{d}{2})$ and $G(\frac{d}{2}) = 3 - \frac{1}{4}(\frac{d}{2})^2$, n is the spin dimensionality. For a three dimensional material ($d = 3$), we have $J(r) \approx r^{-(3+\sigma)}$. When $\sigma \geq 2$, the Heisenberg model ($\beta = 0.365$, $\gamma = 1.386$ and $\delta = 4.8$) is valid for the three dimensional isotropic magnet, where $J(r)$ decreases faster than r^{-5} . When $\sigma \leq 3/2$, the mean-field model ($\beta = 0.5$, $\gamma = 1.0$ and $\delta = 3.0$) is satisfied, expecting that $J(r)$ decreases slower than $r^{-4.5}$. From Eq. (9) $\sigma = 1.908 \pm 0.007$ is generated for FeGe, thus close to the short-range magnetic coupling of $\sigma \sim 2$. Subsequently, it is found that the magnetic exchange distance decays as

$J(r) \approx r^{-4.9}$, which indicates that the magnetic coupling in FeGe is close to a short-range interaction. Moreover, we get the correlation length critical exponent $\nu = 0.709 \pm 0.008$ (where $\nu = \gamma/\sigma$, $\xi = \xi_0|(T - T_C)/T_C|^{-\nu}$), and $\alpha = (2 - \nu d) = -1.127 \pm 0.008$. Theory gives that $\alpha = -0.115(9)$ for 3D-Heisenberg model and $\alpha = 0.110(5)$ for 3D-Ising model [39, 40]. Therefore, these critical exponents indicates that the critical behavior in FeGe is close to the 3D-Heisenberg model with short-range magnetic coupling. However, the discrepancy of the critical exponents to 3D-Ising or 3D-XY models indicates an anisotropic magnetic exchange interaction.

As can be seen from Table I, the critical exponents of $\text{Fe}_{0.8}\text{Co}_{0.2}\text{Si}$ and Cu_2OSeO_3 , which also exhibit a helimagnetic and skyrmion phase transition with similar crystal symmetry, are close to the universality class of the 3D-Heisenberg model [41, 42], indicating a isotropic short-range magnetic coupling. However, the critical behavior of MnSi belongs to the tricritical mean field model [43, 44]. In macroscopic view, the magnetic ordering in cubic FeGe is a DM spiral similar to the structure observed in the isostructural compound MnSi [13]. However, in microscopic view, the magnetic coupling types in these two helimagnets are different. The critical behavior of FeGe is roughly similar to those of $\text{Fe}_{0.8}\text{Co}_{0.2}\text{Si}$ or Cu_2OSeO_3 , except a magnetic exchange anisotropy. In MnSi the spiral propagates are along equivalent $<111>$ directions at all temperatures below $T_C = 29.5$ K. However, it has been revealed that the helical axis (q -vector direction) in FeGe depends on temperature. It is along the $<001>$ direction below 280 K, and changes to the $<111>$ direction in a lower temperature range at 211 K with the decrease of temperature at zero magnetic field [17]. This unique change of helical axis in FeGe may be correlated with the anisotropy of magnetic exchange in this system, since the magnetic exchange anisotropy also plays an important role in the determination of the spin ordering direction. In addition, it should be expounded that the magnetic exchange anisotropy is essentially different from the magnetocrystalline anisotropy. The magnetocrystalline anisotropy is correlated to the crystal structure, while magnetic exchange anisotropy originates from the anisotropic magnetic exchange coupling **J**.

II. CONCLUSION

In summary, the critical behavior of the near room temperature skyrmion material FeGe has been investigated around T_C . The reliable critical exponents ($\beta = 0.336 \pm 0.004$, $\gamma = 1.352 \pm 0.003$, and $\delta = 5.267 \pm 0.001$) are obtained, which are verified by the Widom scaling law and scaling equations. The magnetic exchange distance is found to decay as $J(r) \approx r^{-4.9}$, which is close to that of 3D-Heisenberg model (r^{-5}). The critical behavior indicate that the magnetic interaction in FeGe is of short-range type with an anisotropic magnetic exchange coupling.

III. METHODS

A polycrystalline B20-type FeGe sample was synthesized with a cubic anvil-type high-pressure apparatus. The detailed preparing method was described elsewhere, and the physical properties were carefully checked [H. Du., *et al.*, Nat. Commun. **6**, 8504 (2015)]. The chemical compositions were determined by the Energy Dispersive X-ray (EDX) Spectrometry as shown in Fig. S1 and Table S I, which shows the atomic ratio of Fe : Ge $\approx 50.52 : 49.48$. The magnetization was measured using a Quantum Design Vibrating Sample Magnetometer (SQUID-VSM). The no-overshoot mode was applied to ensure a precise magnetic field. To minimize the demagnetizing field, the sample was processed into slender ellipsoid shape and the magnetic field was applied along the longest axis. In addition, the isothermal magnetization was performed after the sample was heated well above T_C for 10 minutes and then cooled under zero field to the target temperatures to make sure curves were initially magnetized. The magnetic background was carefully subtracted. The applied magnetic field H_a has been corrected into the internal field as $H = H_a - NM$ (where M is the measured magnetization and N is the demagnetization factor) [A. K. Pramanik, *et al.*, Phys. Rev. B **79**, 214426 (2009)]. The corrected H was used for the analysis of critical behavior.

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V. AUTHOR CONTRIBUTIONS

Y. H. Z. conducted the analyses. L. Z. conducted all of the experiments and wrote the paper. H. F. D., C. M. J., and W. S. W. synthesized the sample. H. H. collected the EDX spectrum. M. G. performed the magnetic measurements. J. Y. F., C. J. Z. and L. P. analyzed the experimental results.

VI. ADDITIONAL INFORMATION

Competing financial interests: The authors declare no competing financial interests.

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TABLE I: Comparison of critical exponents of FeGe with different theoretical models and related materials (MAP = modified Arrott plot; Hall = Hall effect; AC = ac susceptibility; SC = single crystal; PC = polycrystal).

Composition	technique	Ref.	T_C (K)	β	γ	δ
FeGe ^{PC}	MAP	This work	283	0.336±0.004	1.352±0.003	5.267±0.001
3D-Heisenberg	theory	[25]	-	0.365	1.386	4.8
3D-XY	theory	[25]	-	0.346	1.316	4.81
3D-Ising	theory	[25]	-	0.325	1.24	4.82
Tricritical mean-field	theory	[28]	-	0.25	1.0	5.0
Mean-field	theory	[25]	-	0.5	1.0	3.0
MnSi ^{SC}	MAP	[44]	30.5	0.242±0.006	0.915±0.003	4.734±0.006
Fe _{0.8} Co _{0.2} Si ^{PC}	Hall	[41]	36.0	0.371±0.001	1.38±0.002	4.78±0.01
Cu ₂ OSeO ₃ ^{SC}	AC	[42]	58.3	0.37(1)	1.44(4)	4.9(1)

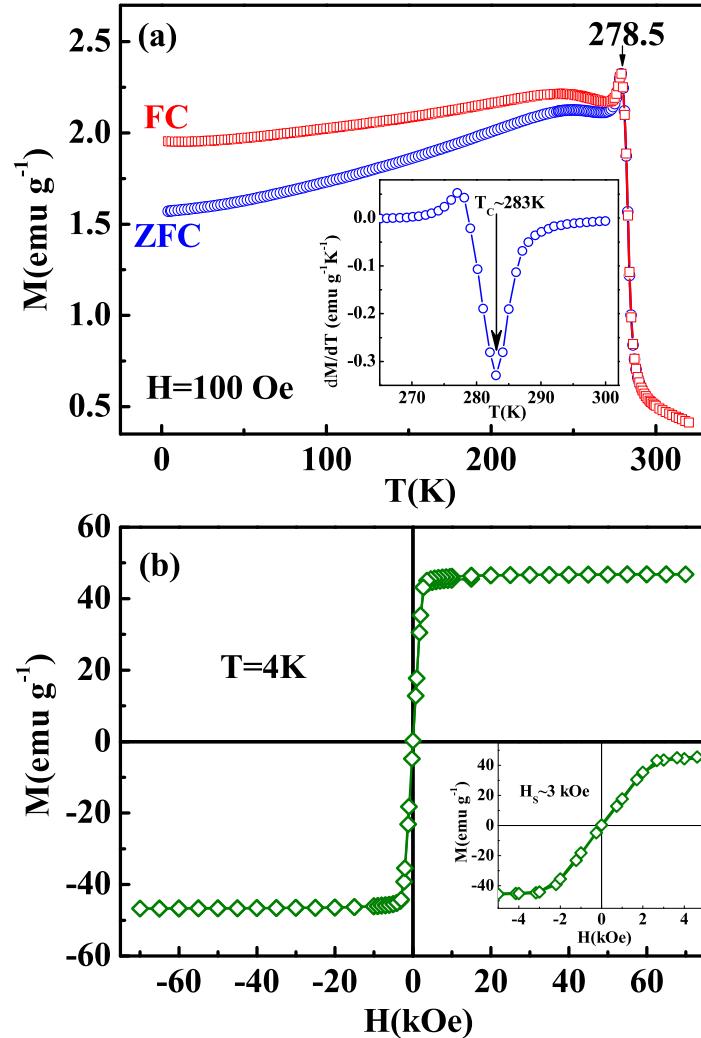


FIG. 1: (Color online) (a) The temperature dependence of magnetization [$M(T)$] for FeGe under $H = 100$ Oe [the inset shows the derivative magnetization (dM/dT) vs T]; (b) the isothermal magnetization [$M(H)$] at 4 K (the inset gives the magnified region in the lower field regime).

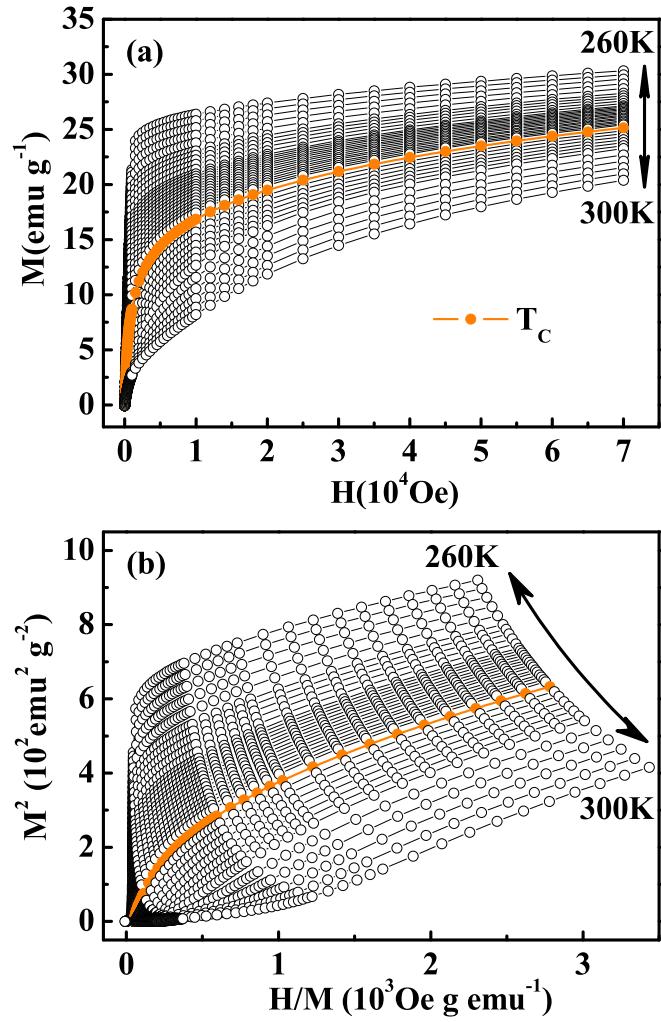


FIG. 2: (Color online) (a) The initial magnetization around T_C for FeGe; (b) Arrott plots of M^2 vs H/M [the $M(H)$ curves are measured at interval $\Delta T = 1$ K, and $\Delta T = 0.5$ K when approaching T_C].

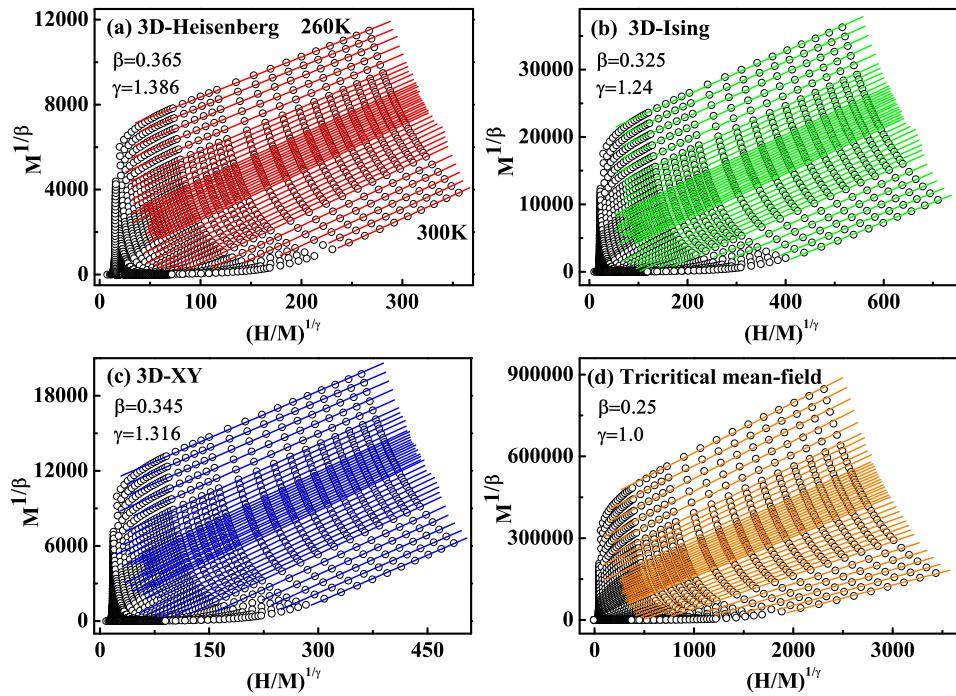


FIG. 3: (Color online) The isotherms of $M^{1/\beta}$ vs $(H/M)^{1/\gamma}$ with (a) 3D-Heisenberg model; (b) 3D-Ising model; (c) 3D-XY model; and (d) tricritical mean-field model.

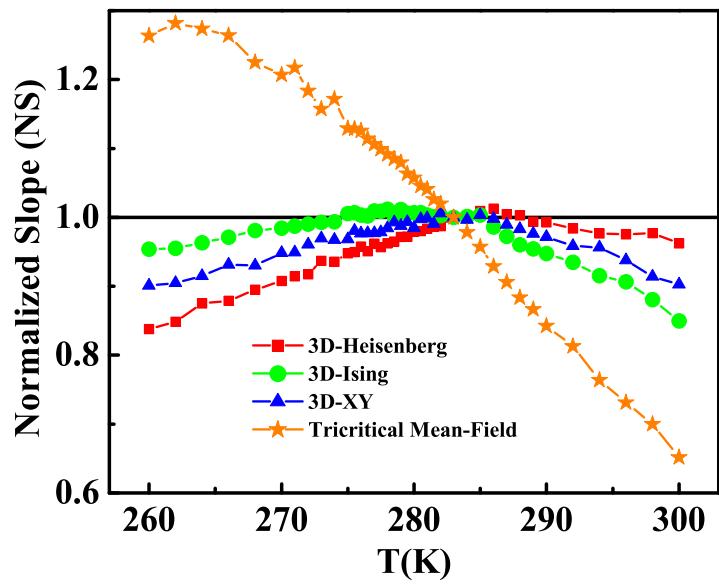


FIG. 4: (Color online) The normalized slopes [$NS = S(T)/S(T_C)$] as a function of temperature.

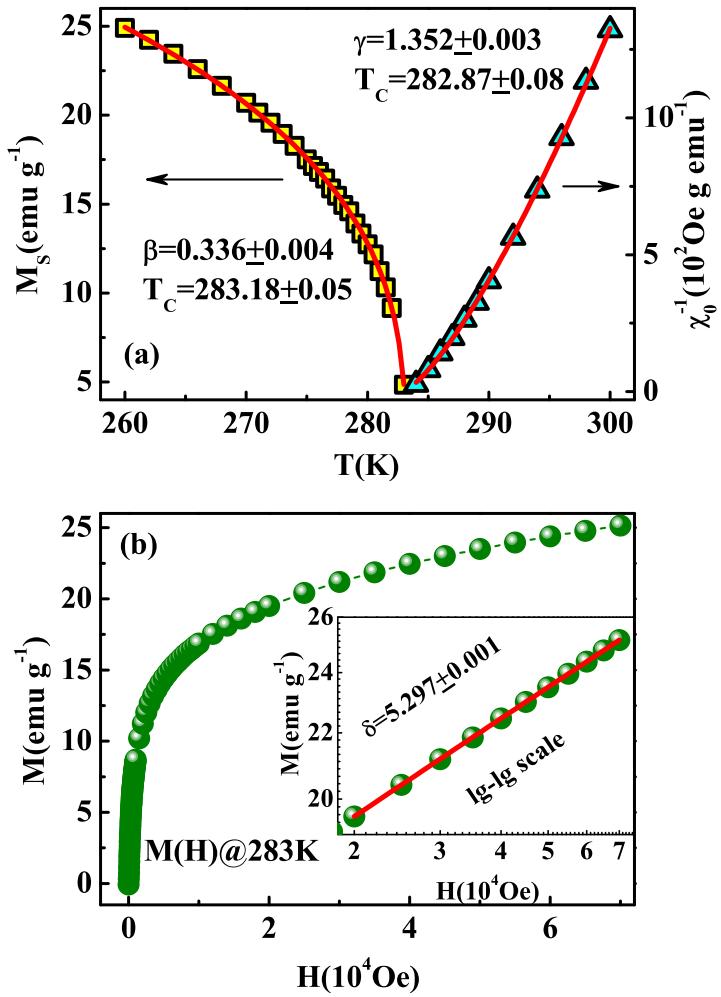


FIG. 5: (Color online) (a) The temperature dependence of M_S and χ_0^{-1} for FeGe with the fitting solid curves; (b) the isothermal $M(H)$ at T_C with the inset plane on lg – lg scale (the solid line is fitted).

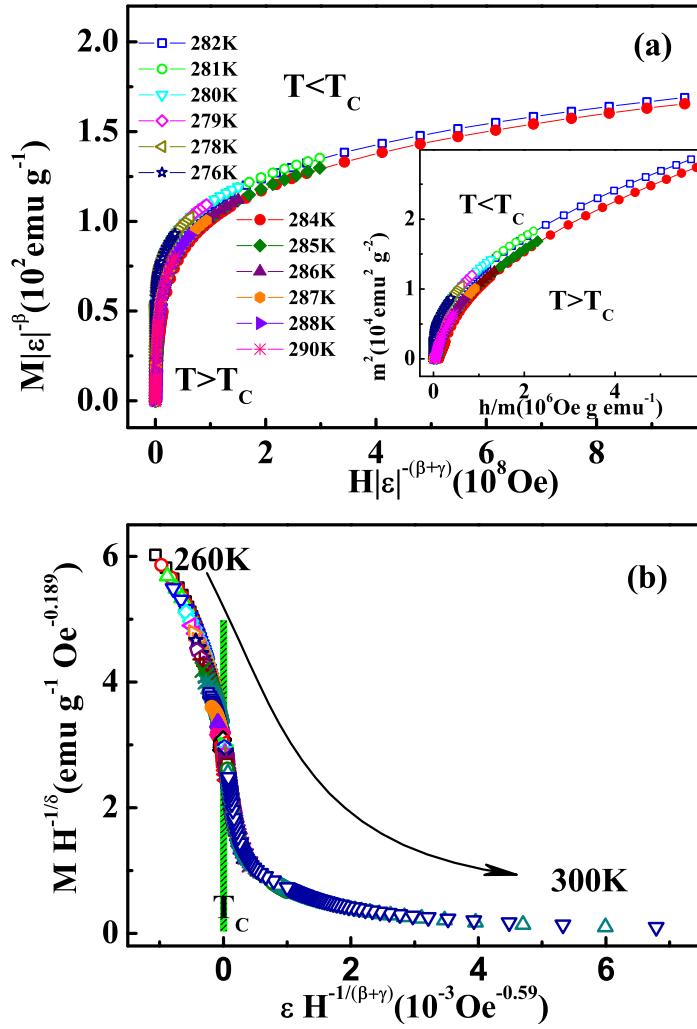


FIG. 6: (Color online) (a) Scaling plots of renormalized magnetization m vs renormalized field h around the critical temperatures for FeGe (the inset shows the m^2 vs h/m); (b) the rescaling of the the $M(H)$ curves by $MH^{-1/\delta}$ vs $\varepsilon H^{-1/(\beta+\gamma)}$.